The Energy Balance Model

D.S. Battisti

Dept. of Atmospheric Sciences, University of Washington, Seattle

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ABSTRACT

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1. Zero-order climatological energy balance model without an absorbing atmosphere

Assume the atmosphere is transparent to the insolation (mainly visible and ultraviolet) and that the energy reaching the planet averaged over a day is

$$S = S_o\Lambda(\mathbf{x},t) \ , \tag{1}$$

where S_o (the solar constant) depends on solar luminosity and distance from the sun and is $\sim$1370 W/m^2 today. $\Lambda(x,t)$ takes into account the angle of incidence as a function of latitude which depends on time of day, time of year, and Milankovich components eccentricity, precession and obliquity. Figure 1 shows the climatological annual cycle of S for Earth, assuming present day orbital parameters.

In the simplest case where all energy arriving at the planet's surface is absorbed, the total absorbed energy is

$$S_o\pi r^2 \ , \tag{2}$$

where r is the radius of the planet. In general, a fraction of the incident energy is reflected back to space. This fraction is called the albedo (α). The fraction of the incident solar energy that is reflected back to space by the aggregate composition of the climate system is called the planetary albedo α_p. Hence, the net energy absorbed by a planet is

$$S_o\pi r^2(1-\alpha_p) \ . \tag{3}$$

In equilibrium, the net energy absorbed must be balanced by the net radiation emitted to space. Assuming that the planet is a perfect blackbody, the radiation to space is

$$\sigma {T_e}^4(4\pi r^2) \ , \tag{4}$$

where T_e is the emitting temperature of the planet (annual, globally averaged) and σ is the Stephan-Boltzman constant, $\sigma = 5.67 * 10^{-8}$ $\mathrm{Wm}^{-2} K^{-1}$. An energy balance therefore requires

$$S_o(1-\alpha_p)/4 \quad = \quad \sigma T_e{}^4 \ . \tag{5}$$

a. Application to Earth

Earth has a planetary albedo of 0.30, of which 0.24 is due to reflections from objects in the atmosphere (mainly clouds) and 0.06 from objects on the surface (mainly ice and snow) (Fig. 2). The annual averaged insolation reaching the top of the atmosphere S_o is 1370 Wm^{-2}. Plugging these numbers into Eq. (5), we find $T_e = 255$ K. The observed surface annual and global averaged surface temperature is ~ 288 K – a difference of 33 K.

2. Energy balance in an absorbing atmosphere

Approximate the atmosphere as a thin radiating slab with temperature T_a and with a selection of greenhouse gases such at the net emission is some fraction ε of that of a blackbody at the same temperature. The net energy emission from the atmosphere is therefore

$$\varepsilon \sigma T_a{}^4 (4\pi r^2) \ , \tag{6}$$

where ε is some bulk emissivity. Taking into account the fact that the slab atmosphere will radiate to both to space and back down to the ground, we can now write the full energy balance for the surface temperature T_s for a more realistic earth with greenhouse gases:

$$S_o(1-\alpha)\,(\pi r^2) \quad + \quad \varepsilon \sigma T_a{}^4\,(4\pi r^2) \quad = \quad \sigma T_s{}^4\,(4\pi r^2) \tag{7}$$

or

$$S_o(1-\alpha)/4 \quad + \quad \varepsilon \sigma T_a{}^4 \quad = \quad \sigma T_s{}^4 \ . \tag{8}$$

The atmosphere in our model gets no direct energy from the Sun, and recognizing that partial emitters are partial absorbers, the atmosphere gets only some of the energy emitted by the ground, so that the energy balance for the atmosphere requires that

$$2\varepsilon\sigma {T_a}^4 \quad = \quad \varepsilon\sigma {T_s}^4 \quad . \tag{9}$$

With no circulation, the solution to Eqs. (8) and (9) is therefore given by

$${T_s}^4 \quad = \quad \frac{S_o(1-\alpha)}{4\sigma(1-\varepsilon/2)} \quad , \quad {T_a}^4 \quad = \quad {T_s}^4/\sqrt{2} \ . \tag{10}$$

With no absorbing atmosphere ($\varepsilon = 0$), the surface temperature T_s would be 255K, or $-18°C$. The observed surface temperature $T_s = 288\ K$ is obtained using a bulk emissivity $\varepsilon = -.76$ and Eq. (8) yield $T_a = 242$ K.[1]

3. Adding circulation

We can create a simple zonally average energy balance model by assuming our EBM holds locally. We can also add a simple treatment of the heat flux convergence D by atmospheric circulation to the steady state the atmospheric energy budget:

$$2\varepsilon\sigma {T_a}^4 \quad = \quad \varepsilon\sigma {T_s}^4 \quad - \quad D \ , \tag{11}$$

where D is the convergence of energy per unit area due to circulation, and the energy balance at the ground is given by Eq. (8). Applied to the zonal average climate, D represents the convergence of energy associated with the meridional energy transport (Fig. 3). The solution to Eqs. (11) and (8) using an idealized approximation to the observed D is shown in Fig. 4.

[1]To be more precise, the global average temperature will be the global average of the local T_e, calculated by replacing S_o in Eq. (5) by $S_o\overline{\Lambda}$, where $\overline{(\ )}$ denotes the climatological average. This yields $T_e = 254\ K$ and $T_s = 286\ K$.

4. The time dependent Energy Budget Model

We can make the EBM time dependent as follows:

$$C_a \frac{\partial T_a}{\partial t} = -2\varepsilon\sigma {T_a}^4 + \varepsilon\sigma {T_s}^4 - D \quad (12)$$

$$C_g \frac{\partial T_s}{\partial t} = S(1-\alpha)/4 + \varepsilon\sigma {T_a}^4 - \sigma {T_s}^4 \ , \quad (13)$$

where $S = S_o\Lambda$ and the left hand sides show the rate of change of energy storage in the atmosphere (Eq. (12)) and "surface "(Eq. (13)) (treating surface as either ocean or dirt). To simplify, these equations can be linearized about a reference temperature $T_r = 273.13K$:

$$\begin{aligned} T_a &= T_r + T_a' \\ T_s &= T_r + T_s' \end{aligned} \quad (14)$$

where $|T_a/T_r|$, $|T_a/T_r| << 1$. Equations (12) and (13) then become

$$C_a \frac{\partial T_a}{\partial t} = -a\,\varepsilon + b\,\varepsilon\,T_s - 2\,b\,\varepsilon\,T_a - D \quad (15)$$

$$C_g \frac{\partial T_s}{\partial t} = S(1-\alpha)/4 - a\,(1-\varepsilon) + \varepsilon\,b\,T_a - b\,T_s \ , \quad (16)$$

where we have dropped the primes and the constants a and b are

$$a = \sigma {T_r}^4 \ ; \quad b = 4a/T_r \ . \quad (17)$$

Plugging in numbers, we find $a = 316\,\mathrm{Wm}^{-2}$ and $b = 4.6\,\mathrm{Wm}^{-2}K^{-1}$.

The heat capacity of the atmosphere is

$$C_a = {C_p}^a \frac{\Delta P}{g} = (10^3\,\mathrm{J\,kg^{-1}K^{-1}}) \quad (10^4\,\mathrm{kg\,m^{-2}}) = 10^7\,\mathrm{J\,m^{-2}K^{-1}} \ . \quad (18)$$

Let's first assume the "ground" is ocean, and we are interested in the seasonal cycle of temperature. In this case, only the near surface ocean (basically the mixed layer) participates and we can estimate the heat capacity of the ground to be

$$C_g = C_o = {C_p}^o\,h\,\rho_{H_2O} \ , \quad (19)$$

where h is the depth of the mixed layer. If we assume the mixed layer depth $h = 75$m, then we find

$$C_g \quad = \quad C_o \quad = \quad (4*10^3)\,(75)\,(10^3) \quad = \quad 3*10^8 \text{ Jm}^{-2}\text{K}^{-1} \;\; . \tag{20}$$

Hence, over the water covered planet, $C_g/C_a = C_o/C_a \approx 30 >> 1$ and thus we can assume the atmosphere is in equilibrium with respect to changes in the "surface" (sea surface) temperature. This allows us to set the left-hand side of Eq. (15) to zero and obtain:

$$0 \quad = \quad -a\varepsilon + b\varepsilon T_s - 2b\varepsilon T_a \quad - \quad D \tag{21}$$

and the equation for the surface temperature Eq. (16) (that is, the sea surface temperature) simplifies to

$$C_o \frac{\partial T_s}{\partial t} \quad = \quad S\,(1-\alpha)/4 \; -A \; - \; B\,T_s \; + \quad D/2 \;\; , \tag{22}$$

where $A = a\;(1-\varepsilon/2) = 195\,\text{Wm}^{-2}$ and $B = b\;(1-\varepsilon/2) = 2.9\,\text{Wm}^{-2}K^{-1}$.

The equilibrium global average ($D = 0$) solution to Eq. (21) and (22) is

$$T_s \;=\; \left(\frac{1370(1-0.3)}{4} - 195\right)/2.9 \quad = \quad 15.4^\circ\text{C} \;\; , \;\; T_a \;=\; -26.6^\circ\text{C} \tag{23}$$

and is independent of the heat capacity of the atmosphere and surface. Eq. (23) is the linearized version of Eq. (10).

It is useful to rearrange Eq. (22) and define the forcing F to be

$$F \equiv S(1-\alpha)/4 - A \quad = \quad C_o \frac{\partial T_s}{\partial t} + B\,T_s \;\; , \tag{24}$$

which has the time dependent solution

$$T_s \quad = \quad e^{-t/\tau} \int_0^t \frac{F(t)}{C_o}\, e^{t/\tau}\, dt \;\; , \tag{25}$$

where

$$\tau \quad = \quad C_o/B \tag{26}$$

is the characteristic response (adjustment) time of the system. For a planet covered by 75m of water, this is about

$$\tau \quad = \quad C_o/B \quad = \quad \frac{3.0\,x\,10^8}{2.9}\,\mathrm{s} \quad = \quad 3.3\,\mathrm{years} \ . \tag{27}$$

By way of contrast, the heat capacity associated with seasonal time scales over land is

$$C_g \quad = \quad C_l \quad = \quad {C_p}^{dirt}\, h_{dirt}\, \rho_{dirt} \; = \; 1200\,\mathrm{J\,kg^{-1}\,K^{-1}} \times 1\,\mathrm{m} \times 2500\,\mathrm{kg\,m^{-3}} \; = \; 3 \times 10^6 \mathrm{J\,m^{-2}\,K^{-1}} \, . \tag{28}$$

For a land covered planet, the adjustment time scale is < 2 weeks.

For an instantaneous switch-on of a constant forcing,

$$F \quad = \quad \left\{ \begin{array}{ll} 0 & t \leq 0 \\ F_o & t > 0 \end{array} \right\} \tag{29}$$

the solution is

$$T_s \quad = \quad \frac{F_o}{B}(1 - e^{-t/\tau}) \ . \tag{30}$$

5. A simple model of the Seasonal Cycle

Subtracting the long term mean from Eq. (22), we find

$$C_o \frac{\partial T_s'}{\partial t} \quad = \quad S' \,(1-\alpha)/4 \; - \; B\, T_s' \ , \tag{31}$$

where prime denotes the deviation from the long term mean. Expanding Eq. (31) in a Fourier series

$$\left\{ \begin{array}{c} S' \\ T_s' \end{array} \right\} \quad = \quad \sum_j \left\{ \begin{array}{c} S_j \\ T_j \end{array} \right\} exp(i\,\omega_j t) \ , \tag{32}$$

the solution is

$$C_o\, i\, \omega_j\, T_{gj} \quad = \quad S_j(1-\alpha)/4 - BT_{gj} \ , \tag{33}$$

where

$$T_j = \frac{S_j(1-\alpha)/4}{B + iC_o\omega_j} \ , \tag{34}$$

or, if S_j is real,

$$T_j = Re\left\{T_j e^{(i\omega_j t)}\right\} = \frac{S_j(1-\alpha)/4}{\left[B^2 + (C_o\omega_j)^2\right]^{1/2}} cos(\omega_j t - \phi_j) \ , \tag{35}$$

where

$$\phi_j = tan^{-1}\left(C_o\omega_j/B\right) \ . \tag{36}$$

a. Annual cycle of Sea Surface Temperature (SST)

For the annual harmonic and over the ocean ($\omega = 2\pi/(\pi\, x\, 10^7)\, s^{-1}$, $C_o = 3 * 10^8$), so

$$\phi_j \sim \pi/2 \tag{37}$$

or $\phi_j = 3$ months. In the midlatitudes, the forcing $S_j/4 \approx 200\,\mathrm{Wm}^{-2}$ (see Fig 1) and so from Eq. (34) we find

$$|T_j| \approx 2°C \ . \tag{38}$$

So the annual cycle in the midlatitude oceans should have amplitude $\pm 2°$C, and lag the maximum in insolation by about 3 months. The observed seasonal harmonic of SST (zonally averaged) is shown in Fig. 7.

The solution to Eq. (34), linearized about the equilibrium solution with heat transport (Eq. (11), is shown in Fig. 5. A more realistic model would take into account general latitudinal dependence of mixed layer depth – deepening in the high latitudes due to vigorous mixing be atmospheric winds and increased buoyancy loss due to wintertime cooling; such a calculation is shown in Fig. 6. In the midlatitudes, this is a fair model of the annual cycle of SST in the Southern Hemisphere, but it underestimates by a factor of three the seasonal cycle in the Northern Hemisphere (Fig. 7).

Large discrepancies also appear near the equator – illuminating the zero order impact of ocean dynamics on the annual cycle of SST along the equator.

b. Seasonal Cycle over land

The seasonal cycle of temperature over land is much greater than the seasonal cycle of SST – mainly because the mass of land that participates is much less than the mass of the ocean because diffusion through a solid is a much less efficient process than mixing of a fluid. Assuming that the heat capacity of the soil that participates in the annual cycle is small compared to the heat capacity of the atmosphere, the equations 15 and 16 are (after removing the time mean climatology)

$$C_a \frac{\partial T_a}{\partial t} = +b\,\varepsilon\,T_s - 2\,b\,\varepsilon\,T_a \tag{39}$$

$$0 = S(1-\alpha)/4 + \varepsilon\,b\,T_a - b\,T_s\ , \tag{40}$$

which has the solution Fourier solution:

$$T_{aj} = \frac{S_j(1-\alpha)/4}{2B + iC_a\omega_j/\varepsilon} \tag{41}$$

$$T_{sj} = \frac{1}{b\varepsilon}\left(i\sigma C_a + 2b\varepsilon\right) T_{aj}, \tag{42}$$

Plugging in numbers, we find the amplitude of the annual cycle of surface temperature over midlatitude land regions to be in excess of $\pm 44°$C and lag insolation by 8 days or so. This is too extreme – largely because we have assumed zero heat capacity. Using a more realistic heat capacity yields the amplitude of the annual cycle of surface temperature of $\pm 30°$C and a lag of 30 days.

6. What is missing?

Lots of things. But the three most important are (i) the atmosphere absorbs some (18%) of the incident solar radiation – mainly by ozone in the stratosphere and water vapor in the troposphere; (ii) turbulent exchange of sensible and latent heat at the surface and (iii) winds advect temperature. The first term is fundamental, albeit unappreciated (the seasonal cycle of temperature in the troposphere is mainly due to absorption of solar energy *in the troposphere*; see Donohoe and Battisti (2013)). The second term mainly acts to amplify (damp) the seasonal cycle in the atmosphere (land) over the land regions, and acts to damp (amplify) the seasonal cycle in the atmosphere (land) over the ocean regions. The final term is also important, as in the midlatitudes westerly winds cause a blending of two fundamentally different end-member solutions discussed in section 5 (see also Donohoe and Battisti (2013); **?**).

References

Donohoe, A., and D. S. Battisti, 2011: Atmospheric and surface contributions to planetary albedo. *Journal of Climate*, **24 (16)**, 4402–4418.

Donohoe, A., and D. S. Battisti, 2013: The seasonal cycle of atmospheric heating and temperature. *Journal of Climate*, **26 (14)**, 4962–4980.

McKinnon, K. A., A. R. Stine, and P. Huybers, 2013: The spatial structure of the annual cycle in surface temperature: Amplitude, phase, and lagrangian history. *Journal of Climate*, **26 (20)**, 7852–7862.

Trenberth, K. E., and J. M. Caron, 2001: Estimates of meridional atmosphere and ocean heat transports. *Journal of Climate*, **14 (16)**, 3433–3443.

LIST OF FIGURES

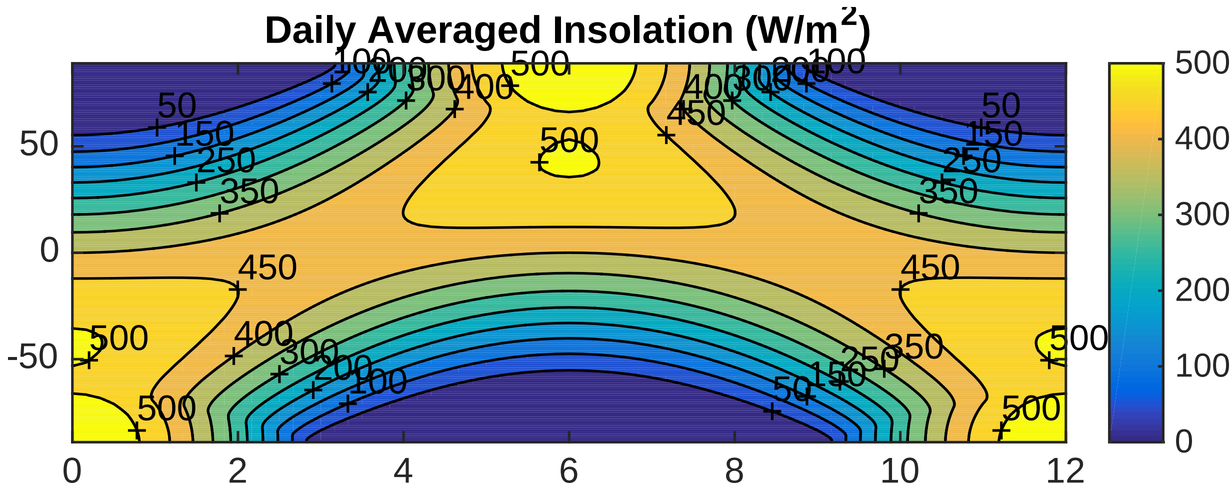

FIG. 1. The top of the atmosphere incoming insolation.

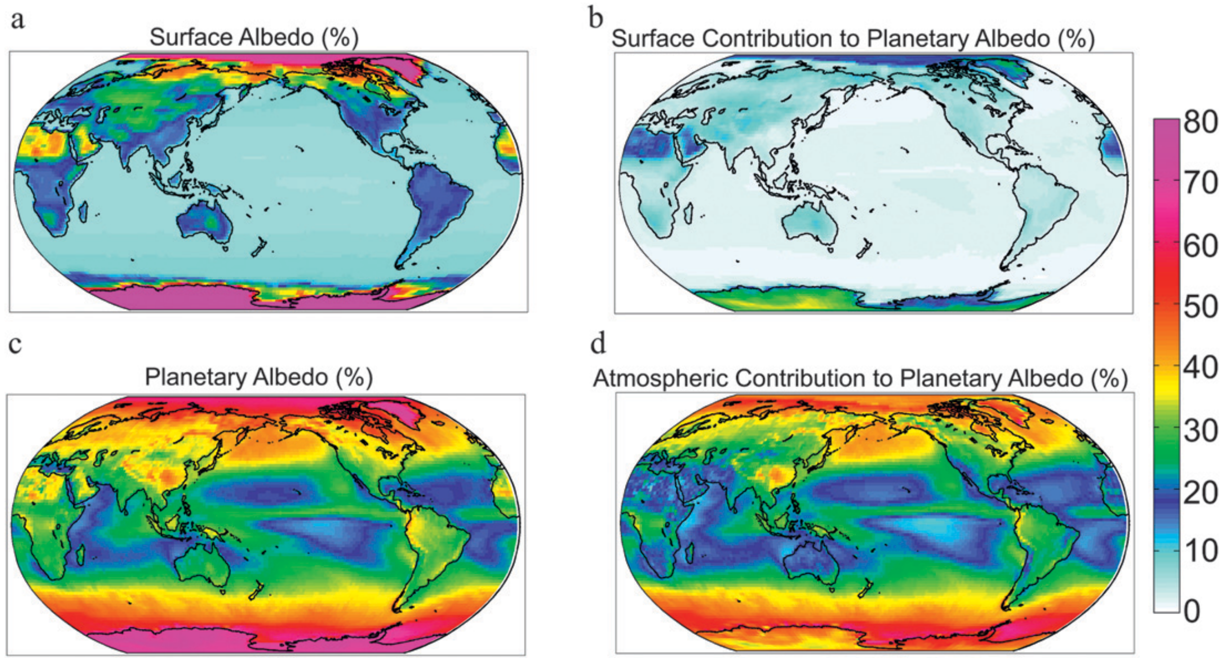

FIG. 2. (a) Surface and (c) planetary albedo, and (b) surface and (d) atmospheric contributions to (c). All quantities are expressed as a percentage, where 1% corresponds to 0.01 units of albedo. From Donohoe and Battisti (2011).

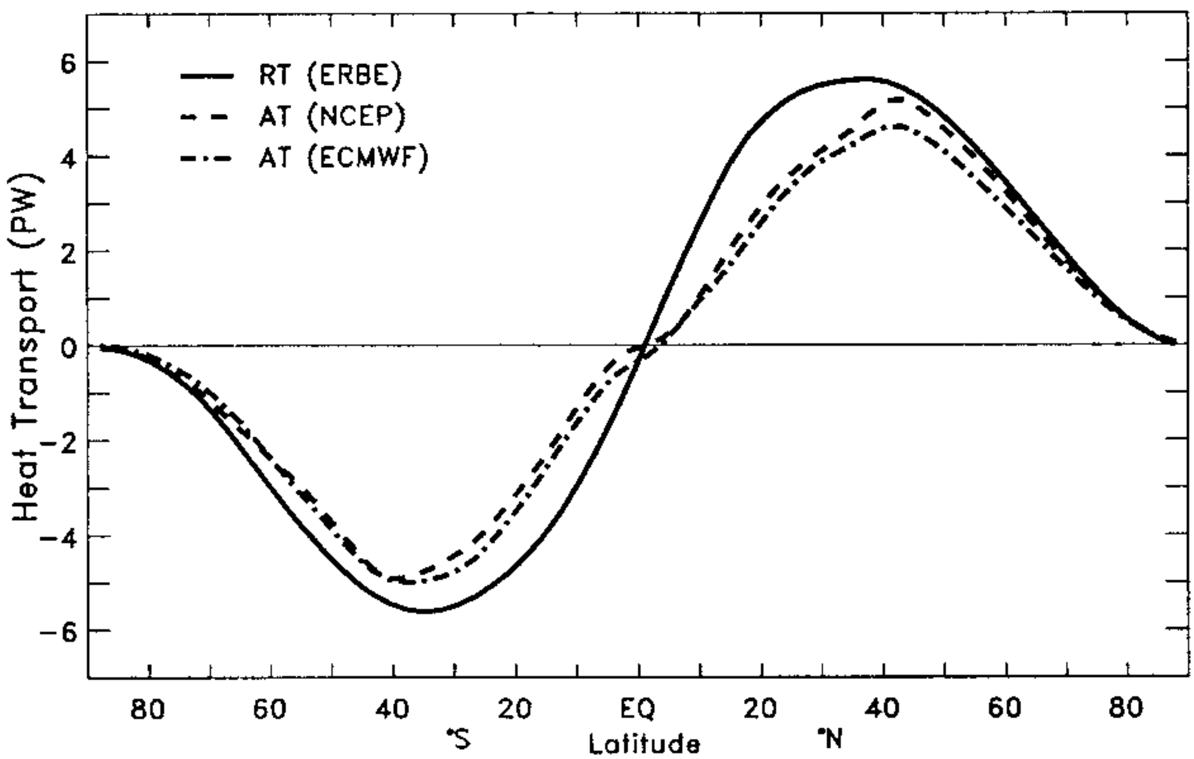

FIG. 3. The climatological annual average northward energy transport. The solid line is the total transport by the ocean and atmosphere, deduced from the top of the atmosphere net radiation and with the assumption that the system is in equilibrium. The dashed lines are estimates of the atmospheric contribution to the total transport, calculated from two reanalysis products. Due to uncertainty in ocean data, the ocean contribution is most reliably estimate by the difference between the the total transport and the atmospheric transport. From Trenberth and Caron (2001).

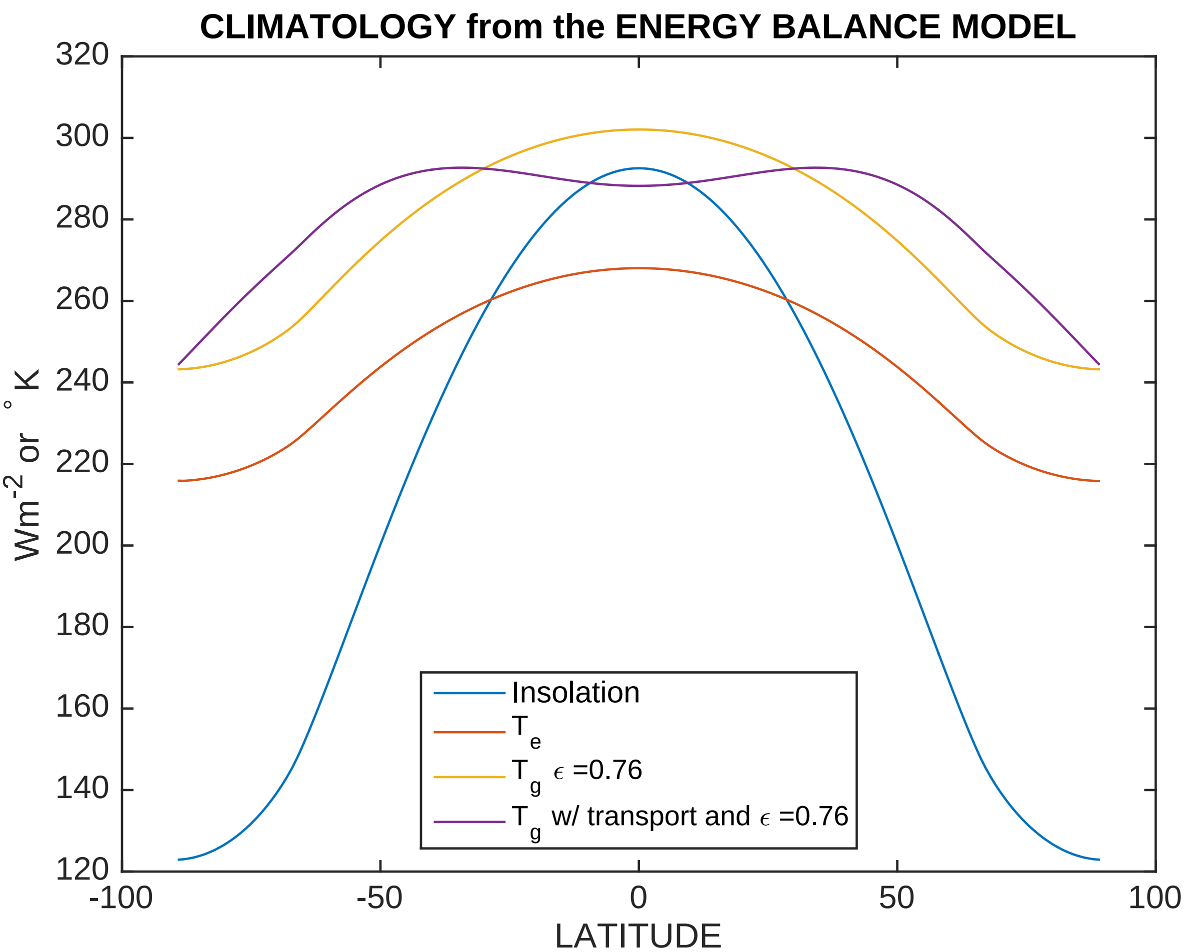

FIG. 4. The zonally and annual average insolation (blue curve) and solutions to the various energy balance models: (red) the emitting temperature (Eq. (5)); (yellow) with an absorbing atmosphere but no circulation (Eq. (10) with $D = 0$); and (purple) with an absorbing atmosphere and circulation ($D = Q_o cos(3\phi)$ in Eq. (11), where $Q_o = -50$ Wm^{-2} and ϕ is latitude). The units are Wm^{-2} and K for insolation and temperature, respectively.

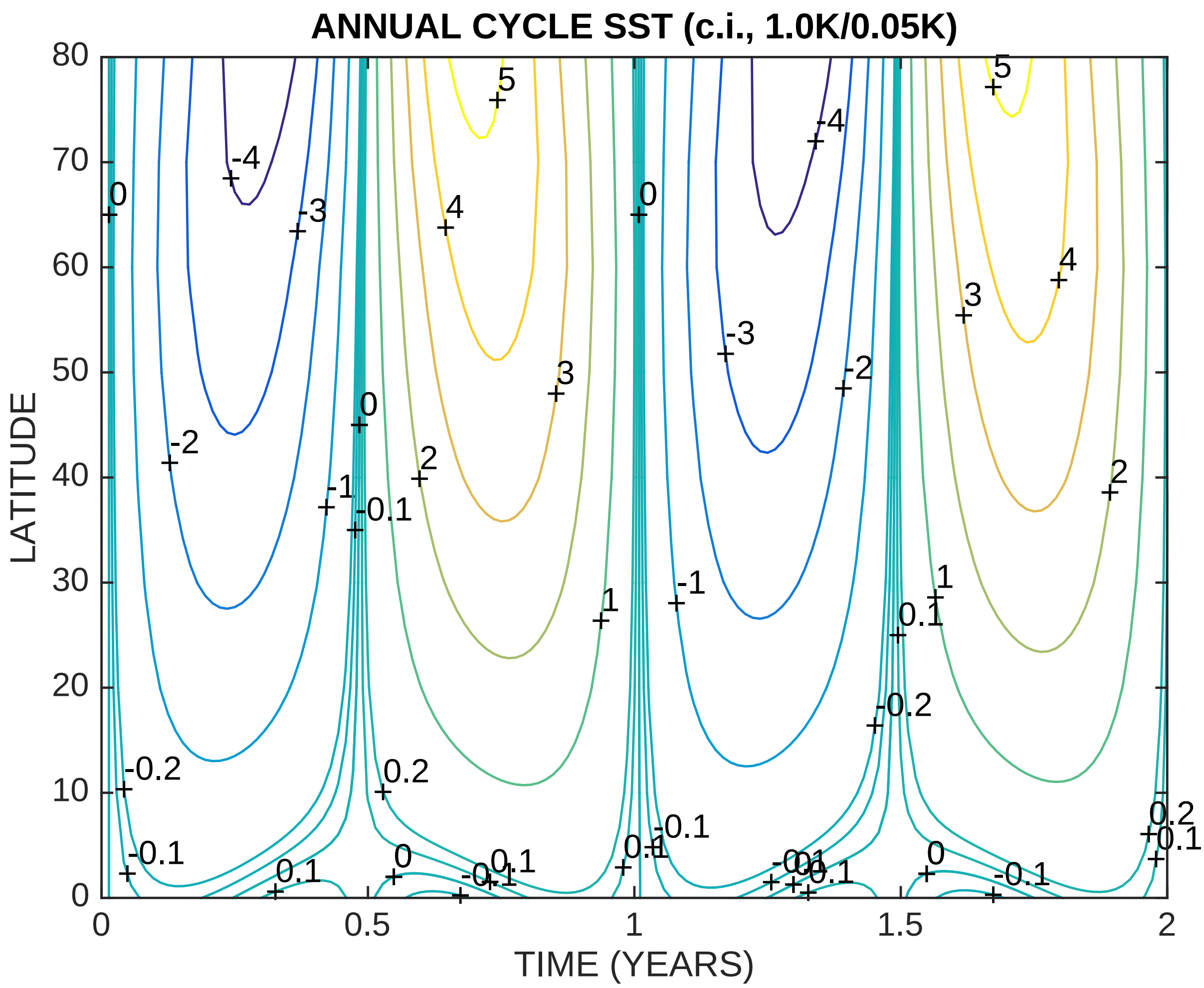

FIG. 5. Solution to the linearized time-dependent energy balance model Eq. (31), forced by the annual cycle of insolation (with the time mean removed). In this calculation, the mixed layer depth is taken to be 75m, $\alpha = 0.3$, $\varepsilon = 0.76$, and the model has been linearized about the nonlinear solution to the equilibrium solution with idealized meridional energy transport is in Fig. 4. Contour interval is ... ,-2, -1, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 1, 2, ... °C.

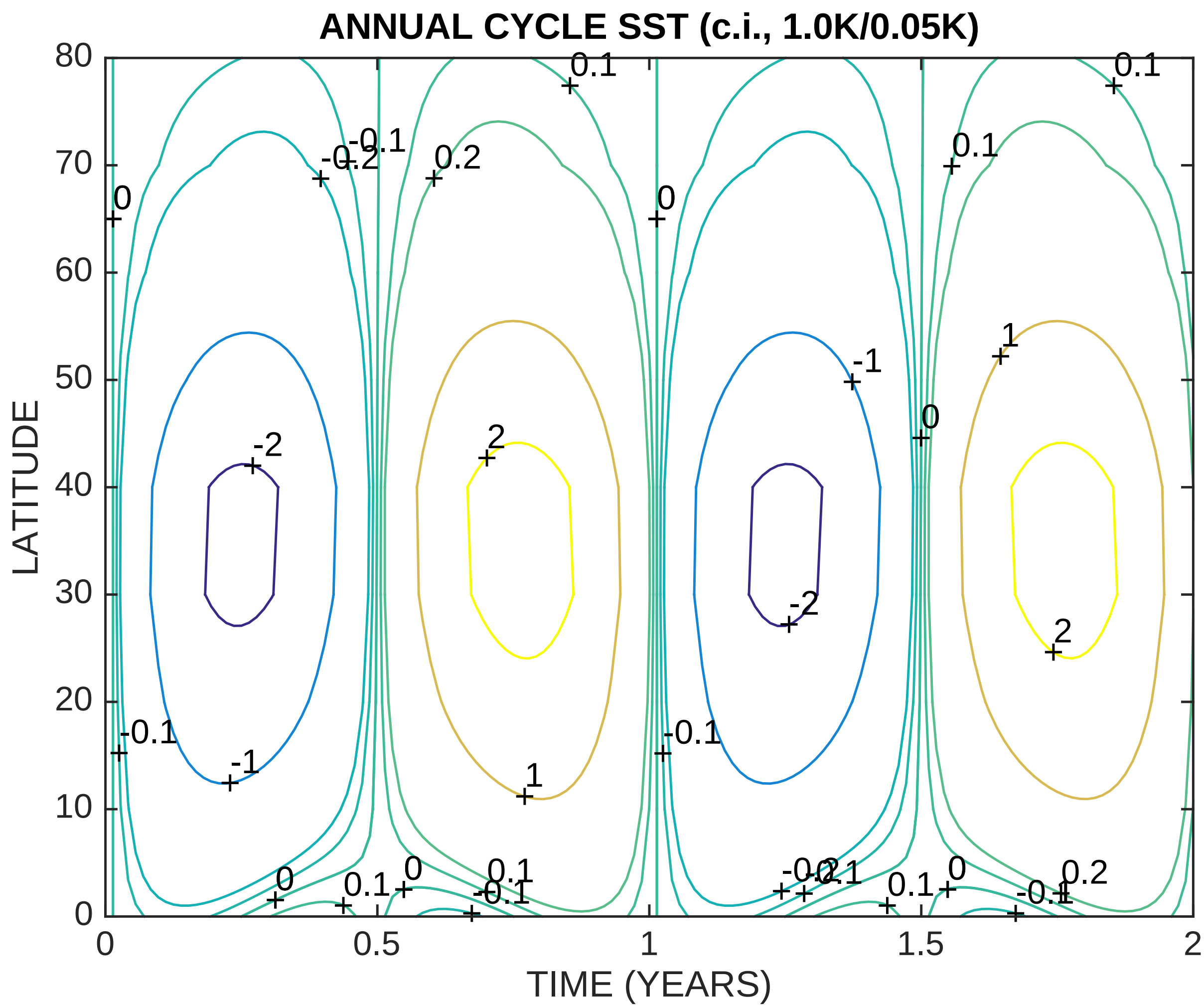

FIG. 6. As in Fig. 5, but including a simple parameterization of mixed layer depth as a function of latitude.

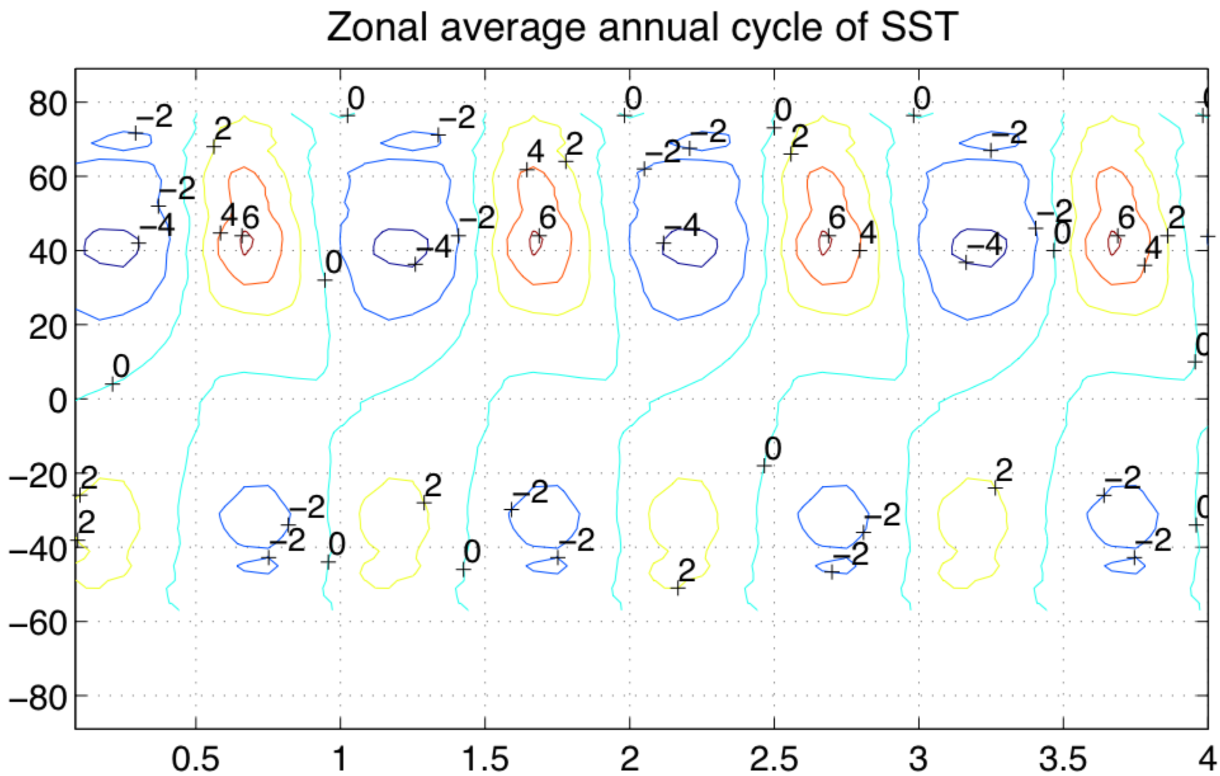

FIG. 7. Observed seasonal cycle zonally averaged SST. Contour interval is 2°C.